

(I) On Point of Inflection

(1) Email from G.Y. dated 20/2/2013

Dear Sir,

I am G. Y. (name omitted for confidentiality) and have just started my career in teaching Mathematics. I found your article while searching sites for information on points of inflection. Because I have gone through a lot many books and no where nobody has discussed about the existence of points of inflection at the points of non differentiability or discontinuity of a function. The basic definition every one gives is "the points where the curve changes its concavity are the points of inflection" and at some places I have read "the points at which extremum occurs cannot be points of inflection".

So I am very much confused about the points at which the the concavity of the curve is changing but also it is a point of extremum and clearly would be non differentiable at that point.

Although your article says that points of inflection can also occur at the points of extremum but I just want to be 100% sure so If you can kindly tell me is there any reference book which can confirm it then that would be great because i have done a lot of research and couldn't find anything that can remove my doubt. I will be waiting for your reply.

Thanks & Regards

(2) My reply email on 21/2/2013

Dear G. Y.,

I am a retired teacher. I am not teaching for 3 - 4 years and I gave most of my textbooks to other teachers and students. Therefore my references may not be too good.

In mathematics, different textbooks (or websites) may employ slightly different definitions for math terms and also slightly different axioms. There are reasons behind this.

- (1) some books tries to avoid too difficult things.
- (2) some books do not like to discuss things outside the scope of the books.
- (3) there are slightly different cultures in mathematics for different countries.

For a simple example: $y = |x|$, the function is not differentiable in $x = 0$. (left and right derivatives are different). However for this v-shaped curve, y is obvious a minimum at $x = 0$. Since this problem is not too "calculus", some books only discuss differentiable functions for max/min.

For point of inflection (inflexion), there is much confusion.

(1) In **wikipedia**, it is "a point on a curve at which the curvature or concavity changes sign from plus to minus or from minus to plus. The curve changes from being concave upwards (positive curvature) to concave downwards (negative curvature), or vice versa." I agree. But then it is "a point where the tangent meets the curve" which implies **the existence of tangent**. I personally disagree. e.g. $y = x^3$, there is a point of inflection at $x = 0$. If we reflect the curve about the line $y = x$, we get $y = x^{1/3}$, the tangent (and the first derivative) does not exist at $x = 0$ and the point of inflection should not disappear after reflection.

(2) In **mathworld**: <http://mathworld.wolfram.com/InflectionPoint.html>

A necessary condition for x to be an inflection point is $f''(x) = 0$. A sufficient condition requires $f''(x + \epsilon)$ and $f''(x - \epsilon)$ to have opposite signs in the neighborhood of x (Bronshtein and Semendyayev 2004, p. 231). <= you may consult this reference

Note that here $f''(x)$ may or may not exist, since we discuss only near-by points between $x - \epsilon$ and $x + \epsilon$. The definition requires only convexity and concavity before and after the point.

In this case, point of inflection can be max/min.

(3) In Hong Kong, AL 1995 Pure Math (II) Q.9, the function $y = \frac{|x|}{(x+1)^2}$, the marking scheme

does not accept $x = -1$ as both minimum and pt of inflection. But in 2003 Paper 2 Q.7,

$y = \frac{x|x+1|}{x+2}$, the marking scheme **accept** $x = -2$ as both minimum and point of inflection. So

there is an inconsistency (or change) in definition.

(4) Textbooks in HK now seem to accept functions to have both max/min and point of inflection by accepting $f''(x)$ can be undefined at the point of interest.

(5) In my point, whether you accept or not depend on what definition of point of inflection you are using. Further reference:

<http://math.stackexchange.com/questions/80655/is-it-possible-to-be-both-a-relative-max-min-and-an-inflection-point>

(6) In teaching, it is good to define well before you carry on with your theorems and examples.

I hope the discussion above may help.

Best regards,

K.C.Yue

(II) On Classification of General Conics

(3) Email from G.Y. dated 22/2/2013

Thanks a lot sir to share your knowledge with me, I am extremely thankful to you for guiding me and clearing my doubts.

At times I find myself confused in some of the areas of mathematics which are grey and I cannot find satisfactory logic from any resources, for example recently I was reading about general equation of 2nd degree and conics:

Pair of straight lines are represented by the general equation of 2nd degree if $\Delta = 0$, and further if $h^2 - ab < 0$ it will represent pair of imaginary lines with real point of intersection. However some conditions can be present in case of point circle for e.g.

$x^2 + y^2 - 2x - 2y + 2 = 0$, so what should we say about the locus of (x, y) following the given equation : pair of straight lines or circle or point?

I hope you won't mind if I trouble you with some of my doubts to gain from your experience and knowledge.

Thanks & Regards

G. Y.

(4) My reply email on 22/2/2013

Hi,

I put down conic section for quite a while. Here is what I get:

Given the general conics : $f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

If $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$, the conic section is degenerated.

Observe that:

1. A circle is a special form of ellipse in which $h = 0$ and $a = b$.
2. A degenerated ellipse (including circle) is a point or empty set.
3. A degenerated hyperbola is a pair of intersecting straight lines.
4. A degenerated parabola is a pair of parallel lines or empty set.

- (i) For $\Delta=0$, $f(x,y)$ is a pair of straight lines (real or imaginary) which in term is a degenerated hyperbola.
- (ii) If Δ not zero, $h^2-ab = 0$, $f(x,y)$ is a parabola,
 $h^2-ab < 0$, $f(x,y)$ is an ellipse,
 $h^2-ab > 0$, $f(x,y)$ is a hyperbola.

Your given circle $x^2 + y^2 - 2x - 2y + 2 = 0$ with $\Delta = 0$ the conic section is degenerated. It can be a pair of intersecting or parallel straight lines, a point or empty set)

Since $h^2-ab < 0$, it is an ellipse (in fact it is a circle) , it becomes a degenerated ellipse, which is a point in this case.

Your equation becomes $(x - 1)^2 + (y - 1)^2 = 0$ or $[(x - 1) + i(y - 1)][(x - 1) - i(y - 1)] = 0$
 You have two imaginary straight lines $(x - 1) + i(y - 1) = 0$ and $(x - 1) - i(y - 1) = 0$ which intersect at real point (1, 1).

There is no contradiction:

- (1) a degenerated ellipse becomes a point which is also
- (2) a degenerated hyperbola \Rightarrow pair of imaginary straight lines intersected at a point.

For your consideration :

An ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is an imaginary ellipse $\frac{x^2}{a^2} - \frac{(iy)^2}{b^2} = 1$

Leave the subject for long. So I may not answer your question.

Bye for now,
 K.C.Yue